GVC Companion Guide: Pre Calculus

COURSE FOCUS: To strengthen students' conceptual understanding of problems and mathematical reasoning in solving problems.

Main topics: complex numbers, rational functions, trigonometric functions and their inverses, inverse functions, vectors and matrices, and parametric and polar curves

- Set classroom norms and excitement for a great year of mathematics ahead! Start the school year with growth mindset work doing Week of Inspirational Math(s) - youcubed. Consider using these tasks throughout the year with a big kick off week one using parts of any of the "weeks" provided. Work with your PLC to collaborate together on which to use when.
- Use number talks/sense making routines and mini-lessons to bring back past math knowledge. Things to think about including:
    - "My Favorite No" – My favorite wrong answer/Error Analysis
    - Which one doesn't belong? (WODB)
    - Which would you rather?
    - Silent Board Game (How To)
    - Graphing Stories: Blog-why-how-samples; Desmos Stories; STEMlearning; sample search
    - Open Middle (open-ended questions)
    - Estimation 180
    - Number-Math Talks | Same but Different
    - Always, Sometimes, or Never | True-False … and why? | Give example(s) and/or counter-example(s) nrich.maths-ASN | true/false routine
    - Academic Talk protocol (English learners) | Partner A/Partner B (Academic Talk protocol)
    - Claims-Evidence Writing (graphic organizer support) | Problem-Evidence-Reasoning-Claim (PERC)
    - Mathematical Mindsets by Jo Boaler (Appendix A pgs. 217-268) Appendix A

Math Core 8 Milestones as background content knowledge and expectations with which students enter Algebra I

- Algebra Warm Ups for Geometry Teachers, and others… ~ Keeping it Fresh!

- PreCalculus Overview Hon PreCalculus Overview

Pre Calculus Pertinent Mathematics Vocabulary: (tier 2-3)

- Function, maxima and minima, inverse, domain, range, intercepts
- Absolute value, Square root
- Linear, Exponential, Quadratic, Cubic, Logarithmic, laws, natural base
- Transformations, multivariable, polynomials
- Matrix (matrices), determinant, multivariable
- Complex numbers, asymptotes, rational functions
- Conics, ellipse, hyperbola, standard form
- Sinusoidal, sine, cosine, tangent, trigonometric, radian, amplitude, phase shift, coterminal, identities
- Vector, magnitude, geometric, velocity
- Parametrics, parametrization
- Polar coordinates and function

Guiding Philosophy, CGI, Cognitively Guided Instruction:

- We invite you to consider the following:
    - Students need space and time to make sense of mathematics.
    - Students need time to explain their thinking.
    - Take time to notice, strategically share, and celebrate diverse student thinking.
    - Use questioning to elicit, support, and extend thinking.
    - Facilitate student-centered discussions to deepen understanding and create spaces for sense-making.

Seven Guiding Principles of Cognitively Guided Instruction, CGI:

We tie these principles to the Social Justice Standards, learning for justice anti-bias framework - [Learning for Justice Website]

1. Every student comes to math class knowing some mathematics
2. Every student is capable of extending their mathematical ideas
3. Knowing the development of children's thinking helps you know how to support learning– "What am I working toward?"
4. Details of children's thinking support instructional decision making
5. Must challenge our assumptions about what students know and are able to do
6. Must create space for the participation of each and honor the different ways in which students are participating
7. Identity shapes participation, so want to position students competently

SEMESTER 1:

Functions and their graphs—polynomials, rational, exponential, logarithmic

Linear systems with Matrices and Conics

SEMESTER 1 **Fall IAB Algebra and Functions II**		
Functions and transformations review; polynomial functions, exponential and log functions, and conics. Performance Tasks: Aquarium and/or Box problem; "Rupprecht's" circle, ellipses, hyperbolas tasks; Exponential functions ~ China vs India Population task from Okla		
Students should build on **prior knowledge** of…	Students should **master**…	Students should be **developing** and will continue to work on…
Linear, Exponential, Quadratic Functions Transformations Systems of linear equations Will be extending Alg II work with: o work with more complicated rational functions—graphing them and determining zeros, intercepts, asymptotes, increase/decrease, max/min o make connections between matrices and transformation of the plane	Describe, analyze, and interpret functions Solve types of functions and perform operations (compositions). Apply systems with these same functions Analyze and investigate polynomial functions Define and analyze conic sections algebraically and graphically Sketch and analyze the graphs of exponential and logarithmic functions Evaluate and solve logarithmic expressions and equations.	Introduced in semester 1 and continued in semester 2: o Transformations o Unit Circle o Special Right Triangles o Right Triangle Trig o Laws of Sines and Cosines o Complex Numbers o Solving Quadratic type problems o Factoring o Mathematical proofs Knowing if an answer is reasonable

Fall, October, District IAB will be Algebra and Functions II – Quadratic Functions, Equations, and Inequalities

Things to be aware of:

- Need to be sure that students have a clear understanding of transformations of functions (analyzing graphs in general)
- Second semester begins with right triangle trigonometry and sinusoidal
- Matrices are in semester 1
- Working towards being able to determine and to find an appropriate model for a real world application
- Working towards being more fluent with graphing calculators
- Working towards being able to think more abstractly

SEMESTER 2:
Functions: Trigonometric, Parametric, Polar
Analytic Trigonometry and Vectors
Complex Numbers

SEMESTER 2 **IAB Geometry Measurement and Modeling**
Sinusoidal function application Performance Tasks: Bay of Fundy task

Students should build on **prior knowledge** of…	Students should **master**…	Students should be **developing** and will continue to work on…
Will be extending Alg II/Geometry work: o Build from parabolas and circles. Now include ellipses and hyperbolas o Complex numbers—see that they can be represented in the Cartesian plane and note geometric interpretation **o** Extend work with trigonometric functions while investigating reciprocal/inverse functions	Solve trigonometric equations Graph trig functions Work with sinusoidal applications Verify using trig identities	Graphing on alternative planes (polar, complex, parametric) Convert from rectangular to polar and back Graph polar equations

Winter, February District IAB will be Geometry Measurement and Modeling

Things to be aware of:

- Students need to be able to develop the unit circle but no need to memorize it
- Students should have experiences developing trig identities but no need to memorize
- Consider Physics problems during the Vectors unit
- Polar Functions unit comes before Parametric unit

Pre Calculus incorporates many standards from the Conceptual Category: Modeling
Specific standards for higher mathematics are marked with a★symbol to indicate they are modeling standards. Modeling at the higher mathematics level goes beyond the simple application of previously constructed mathematics and includes real-world problems. True modeling begins with students asking a question about the world around them, and the mathematics is then constructed in the process of attempting to answer the question. When students are presented with a real-world situation and challenged to ask a question, all sorts of new issues arise (e.g., Which of the quantities present in this situation are known and which are unknown?). Students need to decide on a solution path that may need to be revised. They make use of tools such as calculators, dynamic geometry software, or spreadsheets. They try to use previously derived models (e.g., linear functions), but may find that a new equation or function will apply. Additionally, students may see when trying to answer their question that solving an equation arises as a necessity and that the equation often involves the specific instance of knowing the output value of a function at an unknown input value.
Modeling problems have an element of being genuine problems, in the sense that students care about answering the question under consideration. In modeling, mathematics is used as a tool to answer questions that students really want answered. Students examine a problem and formulate a *mathematical model* (an equation, table, graph, and the like), compute an answer or rewrite their expression to reveal new information, interpret and validate the results, and report out. These efforts should show students that mathematics is relevant to their lives. From a pedagogical perspective, modeling gives a concrete basis from which to abstract mathematics and often serves to motivate students to become independent learners.

For the Pre Calculus course, instructional time should focus on four critical areas (Honors PreCalc):
(1) Functions. (2) Trigonometry (3) Analytic Geometry (4) Linear Systems using Matrices

(1) While many of the standards for **functions** appeared in previous courses, students now apply them in cases of polynomials of degree greater than two, more complicated rational functions, and exponential or logarithmic functions. Students examine the end behavior of these functions and learn to find asymptotes. In addition, students will analyze functions using different representations.

(2) Students will expand their understanding of **trigonometric functions** using the unit circle. They will model periodic phenomena with trigonometric functions, prove and apply trigonometric identities, and apply trigonometry to triangles (law of sines/cosines, vectors, trigonometric form of complex numbers).

(3) Students derive the **equations of conics** (circles, parabolas, ellipses, and hyperbolas) and translate between their graphs and equations. Students work with parametrics, converting to Cartesian form. They understand polar coordinates and the graphs of **polar functions** (circles, cardioids, limacons, roses).

(4) Students expand their knowledge of **linear systems by solving application problems using matrices**.

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Standards for Mathematical Practices = the how-to of the content standards

MP1: Make sense of problems and persevere in solving them
MP2: Reason abstractly and quantitatively
MP3: Construct viable arguments and critique the reasoning of others
MP4: Model with mathematics
MP5: Use appropriate tools
MP6: Attend to precision
MP7: Look for and make use of structure
MP8: Look for and express regularity in repeated reasoning

Mathematical Practices 1-3-6 = connections to EL/ELD and NGSS standards