Precalculus

Unit#1: Functions & Their Graphs (2 Weeks)

Goal: Describe, analyze, and interpret graphs of functions.

-Analyze graphs to determine domain and range, zeros, local maxima and minima.
-Recognize graphs and transformations of common functions.
-Sketch the graph of a transformation.
-Use knowledge of graphical symmetry to determine if a function is even, odd or neither
-Identify and graph linear, absolute value, square root, quadratic, cubic and piecewise functions.
-Perform combinations and compositions of multiple functions.
- Find the inverse of a function algebraically and graphically.

Vocabulary: function, maxima, minima, domain, range, transformations, absolute value, square root, symmetry, inverse, intercepts, linear, exponential, cubic, quadratic

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Critical Areas & Content

I) General Forms of Linear Equations
- Summary of Equations of Lines Section 1-1 Page 8 - 9

II) Library of Parent Functions & Domain & Range
- Section 1- 2
- Section 1- 3

III) Analyzing Graphs of Functions
- Section 1-3

IV) Piecewise Functions
- Section1-3

V) Transformations
- Section 1 – 4

VI) Combinations & Compositions
- Section 1- 5

VII) Inverse Functions
- Section 1-6

Assessment/Links/Resources

- **Performance Task:** Hand Problem to introduce patterns and functions
- ****Common Task is Illustrative Math Medieval Archer**

Content Standards

Interpret functions that arise in applications in terms of the context

F-IF 5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★

F-IF 4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★

Building a function that models a relationship between two quantities.

F-BF 3 Identify the effect on the graph of replacing *f(x)* by *f(x) + k, k f(x), f(kx),* and *f(x + k)* for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

F-BF 4 Find inverse functions.
b. (+) Verify by composition that one function is the inverse of another.
c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
d. (+) Produce an invertible function from a non-invertible function by restricting the domain.

Unit#2: Linear Systems & Matrices (3 Weeks)

Goal: Demonstrate the ability to represent systems of equations with matrices, and preform elementary row operations, both algebraic and with technology. To use a problem-solving approach with matrices.

- Solving multivariable linear systems analytically.
-Converting a linear system in Matrix form and vise versa.
-Understand and perform matrix operations.
-Finding the inverse of a square matrix.
-Finding the determinate of a square matrix.
- Use matrices and the determinate to solve real-world problems.

Vocabulary: matrix, matrices, sqare matrix, determinant, multivariable, inverse

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Critical Areas & Content

I) Multivariable Linear Systems
- Section 7-3

II) Matrices and Systems of Equations
- Section 7-4

III) Operations with Matrices
- Section 7-5

IV) The Inverse of a Square Matrix
- Section 7-6

V) The Determinant of a Square Matrix
- Section 7-7

VI) Applications of Matrices
- Section 7-8

Assessment/Links/Resources
- **common formative assessment/ task on "**polynomial box application problem**"

Content Standards

Perform operations on matrices and use matrices in applications

N-VM 6 (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
N-VM 7 (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
N-VM 8 (+) Add, subtract, and multiply matrices of appropriate dimensions.
N-VM 9 (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
N-VM 10 (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
N-VM 11 (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
N-VM 12 (+) Work with 2 × 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

Unit#3: Polynomial & Rational Functions (3 Weeks)

Goal: Demonstrate the ability to use a problem-solving approach to investigate polynomial functions and equations, both with and without the use of technology.

-Determine domain and range, zeros, local maxima and minima, intervals where the graphs are increasing and decreasing, and end behavior.
-Use common characteristics of a polynomial function to sketch the graph.
-Analyze a function numerically and graphically to determine if the function is odd, even, or neither.
-Use the Fundamental Theorem of Algebra to determine the number of zeros of a polynomial.
- Find all rational, irrational, and complex zeros of a polynomial using algebraic methods.
- Use polynomial and rational functions to model and to solve real-world problems.
-Algebraically identify intercepts, holes, and asymptotes in order to sketch graphs of rational functions.
- Use graphical and algebraic methods to solve rational equations.

Vocabulary: complex numbers, asymptotes, rational functions

Critical Areas & Content

I) Quadratic Functions
- Section 2-1

II) Polynomial Functions of Higher Degree
- Section 2-2

III) Real Zeros of Polynomial Functions
- Section 2-3

IV) Complex Numbers
- Section 2-4

V) The Fundamental Theorem of Algebra
- Section 2-5

VI) Rational Functions & Asymptotes
- Section 2-6

VII) Graphs of Rational Functions
- Section 2-7
- Section 2-8 (application problems)

Assessment/Links/Resources
- **Performance Task:** Box Problem

Content Standards

F-IF 7 **Interpret functions that arise in applications in terms of the context**

F-IF 7d (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior

Perform arithmetic operations with complex numbers

N-CN 3 (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

Unit#4: Conics (3 Weeks)

Goal: Demonstrate the ability to define and analyze conic sections algebraically and graphically.
To use a problem-solving approach to investigate conic sections.

-Define and write the equations of parabolas, circles, ellipses, and hyperbolas in standard form.
- Analyze and sketch parabolas, circles, ellipses, and hyperbolas.
-Given a quadratic equation in general form complete the square to write it in standard form.
-Use conic sections to model and solve real –world problems.

Vocabulary: conics, ellipse, hyperbola, standard form

Critical Areas & Content

I) Circles & Parabolas
- Section 9-1

II) Ellipses
- Section 9-2

III) Hyperbolas
- Section 9-3

IV) Application problems

Assessment/Links/Resources
- **Performance Task:** Sinusoidal Wave
- Formative Assessment on Conics designed by Samohi Torres

Content Standards

Translate between the geometric description and the equation for a conic section.

G-GPE 3 (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

G-GPE 3.1 **Given a quadratic equation of the form ax2 +by2 + cx + dy+ e = 0, use the method for completing the square to put the equation in standard form; identify whether the graph of the equation is a circle, parabola, ellipse, or hyperbola, and graph the equation. CA**

Unit#5: Exponential & Logarithmic Functions (2 Weeks)

Goal: Demonstrate the ability to investigate exponential and logarithmic functions and solve real-world problems, both with and without the use of technology.

-Sketch and analyze exponential and logarithmic functions and their transformations.
- Understand the inverse relationship between exponents and logarithms.
-Define the natural base.
-Evaluate logarithms to any base with and without a calculator.
-Use and apply the laws of logarithms and the change of base formula.
-Solve exponential and logarithmic equations.

Vocabulary: exponential, logarithmic, laws, transformations, natural base

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Critical Areas & Content

I) Exponential Functions & Their Graphs
- Section 3-1

II) Logarithmic Functions & Their Graphs
- Section 3-2

III) Properties of Logarithms
- Section 3-3

IV) Solving Exponential & Logarithmic Equations
- Section 3-4

V) Exponential & Logarithmic Models
- Section 3-5

Assessment/Links/Resources

Content Standards

Interpret functions that arise in applications in terms of the context

F-IF 7e Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

Unit#6: Trigonometric Functions (4 Weeks)

Goal: Define trigonometric ratios and apply them to triangle problems in real-world applications. Sketch, graph, and analyze trigonometric functions. Use a problem-solving approach to investigate trigonometric functions and equations, both with and without the use of technology.

-Define and evaluate the six trigonometric ratios.
- Solve triangles using trigonometric ratios.
-Use triangle trigonometry to model and solve real-world problems, including angles of elevation and depression, and indirect measurement.
- Define radian measure and convert angle measures between degrees and radian, including revolutions.
- Graph the six trigonometric functions.
- Identify the domain and the range of basic trigonometric functions.
-Sketch transformations of the sine and cosine functions.
-Identify and sketch the period, amplitude, phase shift, zeros, and zeros of sinusoidal functions.
- Graph and analyze inverse sine, cosine, and tangent.
-Use trigonometric graphs to model and to solve real- world problems.

Vocabulary: trigonometric, sine, cosine, tangent, amplitude, sinusoidal, radian, phase shift

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Critical Areas & Content

I) Radian and Degree Measure
- Section 4-1

II) Trigonometric Functions: The Unit Circle
- Section 4-2

III) Right Triangle Trigonometry
- Section 4-3

IV) Trigonometric Functions of Any Angle
- Section 4-4

V) Graphs of Sine and Cosine Functions
- Section 4-5

VI) Graphs of Other Trigonometric Functions
- Section 4-6

VII) Inverse Trigonometric Functions
- Section 4-7

VIII) Application Problems
- Section 4-8

Assessment/Links/Resources
- **** Sometimes, Always, Never True Unit Circle Trig**
- **Formative Assessment "Ferris Wheel Application" task**

Content Standards

Interpret functions that arise in applications in terms of the context
F-IF 7e Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

Expand the domain of trigonometric functions using a unit circle
F- TF 4 4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Model periodic phenomena with trigonometric functions
F- TF 6 (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
F-TF 7 (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.★

Unit#7: Analytic Trigonometry (3 Weeks)

Goal: Prove and use identities and formulas to solve trigonometric equations.

-Find the measures of coterminal angles.
-Prove and develop basic trigonometric identities.
- Solve trigonometric equations.

Vocabulary: coterminal, identities

Critical Areas & Content

I) Using Fundamental Identities
- Section 5-1

II) Verifying Trigonometric Identities
- Section 5-2

III) Solving Trigonometric Equations
- Section 5-3

IV) Sum and Difference Formulas
- Section 5-4

V) Multiple-Angle and Product-to-Sum Formulas
- Section 5-5

Assessment/Links/Resources

Content Standards

Prove and apply trigonometric identities

F-TF 9 (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.
F-TF 10 Prove the half angle and double angle identities for sine and cosine and use them to solve problems

Unit#8: Vectors & Trigonometry (4 Weeks)

Goal: Demonstrate the ability to use a problem-solving approach in exploring the properties of vectors and their applications.

-Prove The Law of Sines and The Law of Cosines.
- Apply the Law of Sines and The Law of Cosines.
- Define a geometric vector.
- Find the magnitude and the direction of a geometric vector.
-Perform Operations on vectors.
- Use vectors to model and solve real- world problems, including velocity, force, and air navigation.

Vocabulary: vector, magnitude, geometric, velocity

Critical Areas & Content

I) Law of Sines
- Section 6-1

II) Law of Cosines
- Section 6-2

III) Vectors in the Plane
- Section 6-3

Assessment/Links/Resources
- **Performance Task:** Vectors

Content Standards

Apply trigonometry to general triangles.
G- SRT 9 (+) Derive the formula $A = 1/2\ ab\ \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
G-SRT 10 (+) Prove the Laws of Sines and Cosines and use them to solve problems.
G-SRT 11 (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).
Represent and model with vector quantities.
N-VM 1 (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., *v,* |v|,||v||, *v)*.
N-VM 2 (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
N-VM 3 (+) Solve problems involving velocity and other quantities that can be represented by vectors.
Perform operations on vectors.
N-VM 4 (+) Add and subtract vectors.
a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
c. Understand vector subtraction v – w as v + (–w), where –w is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
N-VM 5 (+) Multiply a vector by a scalar.
a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(vx, vy) = (cvx, cvy)$.
b. Compute the magnitude of a scalar multiple cv using $||cv|| = |c|v$. Compute the direction of cv knowing that when $|c|v \neq 0$, the direction of cv is either along $\boldsymbol{v}$ (for $c > 0$) or against $\boldsymbol{v}$ (for $c < 0$).

Unit#9: Parametrics (2 Weeks)

Goal: To express, graph and analyze parametric functions.

-Find a parametrization of a given equation.
- Graph parametric equations and compare to the equivalent Cartesian equation.
- Apply parametric equations to real – world problems

Vocabulary: parametrics, parametrization

Critical Areas & Content

I) Parametric Equations
- Section 9-4

II) Application Problems

Assessment/Links/Resources

Content Standards

Interpret functions that arise in applications in terms of the context

F-IF 10 **(+) Demonstrate an understanding of functions and equations defined parametrically and graph them. CA**

Unit#10: Polar Functions and Complex Numbers (4 Weeks)

Goal: Demonstrate the ability to use a problem-solving approach in exploring the relationships between the complex plane, the Cartesian plane, and the polar coordinate system.

-Plot points using polar coordinates.
-Change Cartesian coordinates to polar coordinates and vice versa.
-Change Cartesian equations to polar equations and vice versa.
-Analyze and graph polar equations.
-Graph complex numbers on the complex plane.
-Find the trigonometric (polar form) form of complex numbers.
-Apply DeMoivre's Theorem to complex numbers.

Vocabulary: polar coordinates and function

Critical Areas & Content

I) Polar Coordinates
- Section 9-5

II) Graphs of Polar Equations
- Section 9-6

III) Trigonometric Form of a Complex Number
- Section 6. 5

Assessment/Links/Resources

Content Standards

Interpret functions that arise in applications in terms of the context
F-IF 11 **(+) Graph polar coordinates and curves. Convert between polar and rectangular coordinate systems. CA**

Represent complex numbers and their operations on the complex plane.
N-CN 4 (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
N-CN 5 (+)Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $(-1 + \sqrt{3}\, i)3 = 8$ *because* $(-1 + \sqrt{3}\, i)$ *has modulus* 2 *and argument* 120°.
N- CN 6 (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.