HONORS PRECALCULUS/CALCULUS A CRITICAL AREAS

Number and Quantity

The Complex Number System

- Perform arithmetic operations with complex numbers.
- Represent complex numbers and operations on the complex plane.

Vector and Matrix Quantities

- Represent and model with vector quantities.
- Perform operations on vectors.
- Perform operations on matrices and use matrices in applications.

Functions

Interpreting Functions

- Build new functions from existing functions.

Trigonometric Functions

- Expand the domain of trigonometric functions using a unit circle.
- Model periodic phenomena with trigonometric functions.
- Prove and apply trigonometric identities.

Geometry

Similarity, Right Triangles, and Trigonometry

- Apply trigonometry to general triangles.
- Applications of circular trigonometric functions to real-world applications

Semesters at a Glance

Semester 1	Semester 2
Unit #1 = Trigonometric Functions Unit #2 = Trigonometric Identities Unit #3 = Conic Sections and Polar Functions Unit # 4 = Parametrics and Vectors	Unit #5 = Limits and Continuity Unit #6 = Derivatives (part I) Unit #7 = Derivatives (part II) Unit #8 = Integrals Unit #9=Matrices

CRITICAL AREAS HONORS PRECALCULUS

(1) Trigonometric Functions

(2) Trigonometry Identities

(3) Polar Functions and Conic Sections

(4) Parametrics and Vectors

Mathematical Practice	Explanation and Examples
MP.1 Make sense of problems and persevere in solving them.	Students expand their repertoire of expressions and functions that can used to solve problems. They grapple with understanding the connection between complex numbers, polar coordinates, and vectors, and reason about them.
MP.2 Reason Abstractly and quantitatively	Students understand the connection between transformations and matrices, seeing a matrix as an algebraic representation of a transformation of the plane
MP.3 Construct viable arguments and critique the reasoning of others	Students continue to reason through the solution of an equation and justify their reasoning to their peers. Students defend their choice of a function to model a real-world situation.
MP.4 Model with mathematics	Students apply their new mathematical understanding to real-world problems. Students also discover mathematics through experimentation and examining patterns in data from real-world contexts.

MP.5 Use appropriate tools strategically	Students continue to use graphing technology to deepen their understanding of the behavior of polynomial, rational, square root, and trigonometric functions.
MP.6 Attend to precision	Students make note of the precise definition of complex number, understanding that real numbers are a subset of the complex numbers. They pay attention to units in real-world problems and use unit analysis as a method for verifying their answers.
MP.7 Look for and make use of structure	Students understand that matrices form an algebraic system in which the order of multiplication matters, especially when solving linear systems using them. They see that complex numbers can be represented by polar coordinates, and that the structure of the plane yields a geometric interpretation of complex multiplication
MP.8 Look for and express regularity in repeated reasoning	Students multiply several vectors by matrices and observe that some matrices give rotations or reflections. They compute with complex numbers and generalize the results to understand the geometric nature of their operations.

Unit #1: Trigonometric Functions (6 Weeks-in two parts)

Goal: Define trigonometric ratios and apply them to triangle problems in real-world applications. Sketch, graph, and analyze trigonometric functions. Use a problem-solving approach to investigate trigonometric functions and equations, both with and without the use of technology.

- Define and evaluate the six trigonometric ratios.
- Solve triangles using trigonometric ratios.
- Use triangle trigonometry to model problems (angles of elevation/depression, indirect measurement).
- Define radian measure and convert angle measures between degrees and radian, including revolutions.
- Graph the six trigonometric functions.
- Identify the domain and the range of basic trigonometric functions.
- Sketch transformations of the sine and cosine functions.
- Identify and sketch the period, amplitude, phase shift, and zeros of sinusoidal functions.
- Graph and analyze inverse sine, cosine, and tangent.
- Use trigonometric graphs to model and to solve real-world problems.

- (4.2) Unit Circle Trig
- (4.4) Trig Values of Any Angle
- (4.5) Sine & Cosine graphs
- (4.6) Tangent, Cotangent, Secant, Cosecant graphs
- (4.7) Inverse Trigonometry Function
- (4.8) Applications to the Real World

Content Standards: F- IF 7e F- TF 4 F- TF 6 F-TF 7

Unit #2: Analytic Trigonometry (4 Weeks)

Goal: Prove and use identities and formulas to solve trigonometric equations.

- Find the measures of coterminal angles.
- Prove and develop basic trigonometric identities.
- Solve trigonometric equations.

I) Using Fundamental Identities Section 5.1
II) Verifying Trigonometric Identities Section 5.2
III) Solving Trigonometric Equations Section 5.3
IV) Sum and Difference Formulas Section 5.4
V) Multiple-Angle and Product-to-Sum Formulas Section 5.5

Content Standards: F-TF 9 F-TF 10

Unit #3: Polar Functions & Complex Numbers (4 Weeks)

Goal: Explore relationships among the complex and Cartesian plane, and the polar coordinate system.

- Plot points using polar coordinates.
- Change Cartesian coordinates and equations to polar form and vice versa.
- Analyze and graph polar equations.
- Graph complex numbers on the complex plane.
- Find the trigonometric (polar form) form of complex numbers.
- Apply DeMoivre's Theorem to complex numbers.

I) Polar Coordinates	Section 9.5
II) Graphs of Polar Equations	Section 9.6
III) Trigonometric Form of a Complex Number	Section 6.5

Content Standards: F-IF 11 N-CN 4 N-CN 5 N- CN 6

---------------------- SEMESTER BREAK -----------------------------

Unit #4 Parametrics and Vectors (2-3 Weeks)

Goal: To express, graph and analyze parametric functions.

- Find a parametrization of a given equation.
- Graph parametric equations and compare to the equivalent Cartesian equation.
- Vectors and Dot Products
- Apply parametric equations to real–world problems.

I) Parametric Equations	Section 9.4
II) Vectors in a Plane	Section 6.3
III) Vectors and Dot Products	Section 6.4

Content Standards: G-SRT 9 G-SRT 10 G-SRT 11 N-VM 1 N-VM 2 N-VM 3 N-VM 4 N-VM 5

AP CALCULUS AB AND BC

Mathematical Practices

Practice 1

Implementing Mathematical Processes 1

Determine expressions and values using mathematical procedures and rules.

Practice 2

Connecting Representations 2

Translate mathematical information from a single representation or across multiple representations.

Practice 3

Justification 3

Justify reasoning and solutions.

Practice 4

Communication and Notation 4

Use correct notation, language, and mathematical conventions to communicate results or solutions.

SKILLS

1.A Identify the question to be answered or problem to be solved *(not assessed)*.

1.B Identify key and relevant information to answer a question or solve a problem *(not assessed)*.

1.C Identify an appropriate mathematical rule or procedure based on the classification of a given expression (e.g., *Use the chain rule to find the derivative of a composite function*).

1.D Identify an appropriate mathematical rule or procedure based on the relationship between concepts (e.g., *rate of change and accumulation*) or processes (e.g., *differentiation and its inverse process, anti-differentiation*) to solve problems.

1.E Apply appropriate mathematical rules or procedures, with and without technology.

1.F Explain how an approximated value relates to the actual value.

2.A Identify common underlying structures in problems involving different contextual situations.

2.B Identify mathematical information from graphical, numerical, analytical, and/or verbal representations.

2.C Identify a re-expression of mathematical information presented in a given representation.

2.D Identify how mathematical characteristics or properties of functions are related in different representations.

2.E Describe the relationships among different representations of functions and their derivatives.

3.A Apply technology to develop claims and conjectures *(not assessed)*.

3.B Identify an appropriate mathematical definition, theorem, or test to apply.

3.C Confirm whether hypotheses or conditions of a selected definition, theorem, or test have been satisfied.

3.D Apply an appropriate mathematical definition, theorem, or test.

3.E Provide reasons or rationales for solutions and conclusions.

3.F Explain the meaning of mathematical solutions in context.

3.G Confirm that solutions are accurate and appropriate.

4.A Use precise mathematical language.

4.B Use appropriate units of measure.

4.C Use appropriate mathematical symbols and notation (e.g., *Represent a derivative using* $f'(x)$, y', *and* $\frac{dy}{dx}$).

4.D Use appropriate graphing techniques.

4.E Apply appropriate rounding procedures.

UNIT 1-THE LIMIT

Enduring Understandings (Students will understand that ...)	Learning Objectives (Students will be able to ...)	Essential Knowledge (Students will know that ...)
EU 1.1: The concept of a limit can be used to understand the behavior of functions.	**LO 1.1A(a):** Express limits symbolically using correct notation. **LO 1.1A(b):** Interpret limits expressed symbolically.	**EK 1.1A1:** Given a function f, the limit of $f(x)$ as x approaches c is a real number R if $f(x)$ can be made arbitrarily close to R by taking x sufficiently close to c (but not equal to c). If the limit exists and is a real number, then the common notation is $\lim_{x \to c} f(x) = R$. **EXCLUSION STATEMENT (EK 1.1A1):** *The epsilon-delta definition of a limit is not assessed on the AP Calculus AB or BC Exam. However, teachers may include this topic in the course if time permits.*
		EK 1.1A2: The concept of a limit can be extended to include one-sided limits, limits at infinity, and infinite limits.
		EK 1.1A3: A limit might not exist for some functions at particular values of x. Some ways that the limit might not exist are if the function is unbounded, if the function is oscillating near this value, or if the limit from the left does not equal the limit from the right. **EXAMPLES OF LIMITS THAT DO NOT EXIST:** $\lim_{x \to 0} \frac{1}{x^2} = \infty$; $\lim_{x \to 0} \sin\left(\frac{1}{x}\right)$ *does not exist*; $\lim_{x \to 0} \frac{\lvert x \rvert}{x}$ *does not exist*; $\lim_{x \to 0} \frac{1}{x}$ *does not exist*
	LO 1.1B: Estimate limits of functions.	**EK 1.1B1:** Numerical and graphical information can be used to estimate limits.

Enduring Understandings (Students will understand that ...)	Learning Objectives (Students will be able to ...)	Essential Knowledge (Students will know that ...)
EU 1.1: The concept of a limit can be used to understand the behavior of functions. *(continued)*	**LO 1.1C:** Determine limits of functions.	**EK 1.1C1:** Limits of sums, differences, products, quotients, and composite functions can be found using the basic theorems of limits and algebraic rules.
		EK 1.1C2: The limit of a function may be found by using algebraic manipulation, alternate forms of trigonometric functions, or the squeeze theorem.
		EK 1.1C3: Limits of the indeterminate forms $\frac{0}{0}$ and $\frac{\infty}{\infty}$ may be evaluated using L'Hospital's Rule.
	LO 1.1D: Deduce and interpret behavior of functions using limits.	**EK 1.1D1:** Asymptotic and unbounded behavior of functions can be explained and described using limits.
		EK 1.1D2: Relative magnitudes of functions and their rates of change can be compared using limits.
EU 1.2: Continuity is a key property of functions that is defined using limits.	**LO 1.2A:** Analyze functions for intervals of continuity or points of discontinuity.	**EK 1.2A1:** A function f is continuous at $x = c$ provided that $f(c)$ exists, $\lim_{x \to c} f(x)$ exists, and $\lim_{x \to c} f(x) = f(c)$.
		EK 1.2A2: Polynomial, rational, power, exponential, logarithmic, and trigonometric functions are continuous at all points in their domains.
		EK 1.2A3: Types of discontinuities include removable discontinuities, jump discontinuities, and discontinuities due to vertical asymptotes.
	LO 1.2B: Determine the applicability of important calculus theorems using continuity.	**EK 1.2B1:** Continuity is an essential condition for theorems such as the Intermediate Value Theorem, the Extreme Value Theorem, and the Mean Value Theorem.

UNIT 2 & 3-THE DERIVATIVE

Big Idea 2: Derivatives

Using derivatives to describe the rate of change of one variable with respect to another variable allows students to understand change in a variety of contexts. In AP Calculus, students build the derivative using the concept of limits and use the derivative primarily to compute the instantaneous rate of change of a function. Applications of the derivative include finding the slope of a tangent line to a graph at a point, analyzing the graph of a function (for example, determining whether a function is increasing or decreasing and finding concavity and extreme values), and solving problems involving rectilinear motion. Students should be able to use different definitions of the derivative, estimate derivatives from tables and graphs, and apply various derivative rules and properties. In addition, students should be able to solve separable differential equations, understand and be able to apply the Mean Value Theorem, and be familiar with a variety of real-world applications, including related rates, optimization, and growth and decay models.

Enduring Understandings (Students will understand that …)	Learning Objectives (Students will be able to …)	Essential Knowledge (Students will know that …)
EU 2.1: The derivative of a function is defined as the limit of a difference quotient and can be determined using a variety of strategies.	**LO 2.1A:** Identify the derivative of a function as the limit of a difference quotient.	**EK 2.1A1:** The difference quotients $\frac{f(a+h)-f(a)}{h}$ and $\frac{f(x)-f(a)}{x-a}$ express the average rate of change of a function over an interval.
		EK 2.1A2: The instantaneous rate of change of a function at a point can be expressed by $\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$ or $\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$, provided that the limit exists. These are common forms of the definition of the derivative and are denoted $f'(a)$.
		EK 2.1A3: The derivative of f is the function whose value at x is $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ provided this limit exists.
		EK 2.1A4: For $y=f(x)$, notations for the derivative include $\frac{dy}{dx}$, $f'(x)$, and y'.
		EK 2.1A5: The derivative can be represented graphically, numerically, analytically, and verbally.
	LO 2.1B: Estimate derivatives.	**EK 2.1B1:** The derivative at a point can be estimated from information given in tables or graphs.

Enduring Understandings (Students will understand that …)	Learning Objectives (Students will be able to …)	Essential Knowledge (Students will know that …)
EU 2.1: The derivative of a function is defined as the limit of a difference quotient and can be determined using a variety of strategies. *(continued)*	**LO 2.1C:** Calculate derivatives.	**EK 2.1C1:** Direct application of the definition of the derivative can be used to find the derivative for selected functions, including polynomial, power, sine, cosine, exponential, and logarithmic functions.
		EK 2.1C2: Specific rules can be used to calculate derivatives for classes of functions, including polynomial, rational, power, exponential, logarithmic, trigonometric, and inverse trigonometric.
		EK 2.1C3: Sums, differences, products, and quotients of functions can be differentiated using derivative rules.
		EK 2.1C4: The chain rule provides a way to differentiate composite functions.
		EK 2.1C5: The chain rule is the basis for implicit differentiation.
		EK 2.1C6: The chain rule can be used to find the derivative of an inverse function, provided the derivative of that function exists.

	LO 2.1D: Determine higher order derivatives.	**EK 2.1D1:** Differentiating f' produces the second derivative f'', provided the derivative of f' exists; repeating this process produces higher order derivatives of f.
		EK 2.1D2: Higher order derivatives are represented with a variety of notations. For $y = f(x)$, notations for the second derivative include $\frac{d^2y}{dx^2}$, $f''(x)$, and y''. Higher order derivatives can be denoted $\frac{d^ny}{dx^n}$ or $f^{(n)}(x)$.

Enduring Understandings (Students will understand that …)	**Learning Objectives** (Students will be able to …)	**Essential Knowledge** (Students will know that …)
EU 2.2: A function's derivative, which is itself a function, can be used to understand the behavior of the function.	**LO 2.2A:** Use derivatives to analyze properties of a function.	**EK 2.2A1:** First and second derivatives of a function can provide information about the function and its graph including intervals of increase or decrease, local (relative) and global (absolute) extrema, intervals of upward or downward concavity, and points of inflection.
		EK 2.2A2: Key features of functions and their derivatives can be identified and related to their graphical, numerical, and analytical representations.
		EK 2.2A3: Key features of the graphs of f, f', and f'' are related to one another.
	LO 2.2B: Recognize the connection between differentiability and continuity.	**EK 2.2B1:** A continuous function may fail to be differentiable at a point in its domain.
		EK 2.2B2: If a function is differentiable at a point, then it is continuous at that point.
EU 2.3: The derivative has multiple interpretations and applications including those that involve instantaneous rates of change.	**LO 2.3A:** Interpret the meaning of a derivative within a problem.	**EK 2.3A1:** The unit for $f'(x)$ is the unit for f divided by the unit for x.
		EK 2.3A2: The derivative of a function can be interpreted as the instantaneous rate of change with respect to its independent variable.
	LO 2.3B: Solve problems involving the slope of a tangent line.	**EK 2.3B1:** The derivative at a point is the slope of the line tangent to a graph at that point on the graph.
		EK 2.3B2: The tangent line is the graph of a locally linear approximation of the function near the point of tangency.

UNIT 4 Contextual Applications of Differentiation

UNIT 4

AP EXAM WEIGHTING	10–15% AB	6–9% BC
CLASS PERIODS	~10–11 AB	~6–7 BC

Contextual Applications of Differentiation

Developing Understanding

Unit 4 begins by developing understanding of average and instantaneous rates of change in problems involving motion. The unit then identifies differentiation as a common underlying structure on which to build understanding of change in a variety of contexts. Students' understanding of units of measure often reinforces their understanding of contextual applications of differentiation. In problems involving related rates, identifying the independent variable common to related functions may help students to correctly apply the chain rule. When applying differentiation to determine limits of certain indeterminate forms using L'Hospital's rule, students must show that the rule applies.

Building the Mathematical Practices

1.D 1.F 2.A 3.F

Students will begin applying concepts from Units 2 and 3 to scenarios encountered in the world. Students often struggle to translate these verbal scenarios into the mathematical procedures necessary to answer the question. To solve these problems, students will need explicit instruction and intentional practice identifying key information, determining which procedure applies to the scenario presented (i.e., that "rates of change" indicate differentiation), stating what is changing and how, using correct units, and explaining what their answer means in the context of the scenario. Provide scenarios with different contexts but similar procedures so students begin to recognize and apply the reasoning behind those problem-solving decisions, rather than grasping at rules haphazardly.

Students must also be able to explain how an approximated value relates to the value it's intended to approximate. Students may not understand why they would use a tangent line approximation (i.e., linearization) rather than simply evaluating a function. Expose them to scenarios where an exact function value can't be calculated, and then ask them to determine whether a particular approximation is an overestimate or an underestimate of the function.

Preparing for the AP Exam

With contextual problems, emphasize careful reading for language such as, "find the rate of change." This will help students understand the underlying structure of the problem, answer the question asked, and interpret solutions in context. Students should not use words like "velocity" when they mean the rate of change in income, for example, even though the underlying structure is the same.

Emphasize that students must verify that $\lim_{x \to a} f(x) = \lim_{x \to a} g(x) = 0$ (or that both approach infinity) as a necessary first step before applying L'Hospital's Rule to determine $\lim_{x \to a} \frac{f(x)}{g(x)}$. Students should understand that $\frac{0}{0}$ or $\frac{\infty}{\infty}$ are appropriate labels for indeterminate forms but do not represent values in an equation. Therefore, it is incorrect to write $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{0}{0}$, for example. Note that $\lim_{x \to a} \frac{f(x)}{g(x)} \neq \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$ when $\lim_{x \to a} g(x) = 0$. Also emphasize that the conclusion of L'Hospital's rule features the ratio of the derivatives of the numerator and denominator, respectively, rather than the derivative of the ratio.

UNIT AT A GLANCE

Enduring Understanding	Topic	Suggested Skills	Class Periods ~10–11 CLASS PERIODS (AB) ~6–7 CLASS PERIODS (BC)
CHA-3	**4.1 Interpreting the Meaning of the Derivative in Context**	1.D Identify an appropriate mathematical rule or procedure based on the relationship between concepts (e.g., rate of change and accumulation) or processes (e.g., differentiation and its inverse process, anti-differentiation) to solve problems.	
	4.2 Straight-Line Motion: Connecting Position, Velocity, and Acceleration	1.E Apply appropriate mathematical rules or procedures, with and without technology.	
	4.3 Rates of Change in Applied Contexts Other Than Motion	2.A Identify common underlying structures in problems involving different contextual situations.	
	4.4 Introduction to Related Rates	1.E Apply appropriate mathematical rules or procedures, with and without technology.	
	4.5 Solving Related Rates Problems	3.F Explain the meaning of mathematical solutions in context.	
	4.6 Approximating Values of a Function Using Local Linearity and Linearization	1.F Explain how an approximated value relates to the actual value.	
LIM-4	**4.7 Using L'Hospital's Rule for Determining Limits of Indeterminate Forms**	3.D Apply an appropriate mathematical definition, theorem, or test.	

Unit#9: Linear Systems & Matrices (3 Weeks)

Goal: Demonstrate the ability to represent systems of equations with matrices, and preform elementary row operations, both algebraic and with technology. To use a problem-solving approach with matrices.

- Solving multivariable linear systems analytically.
-Converting a linear system in Matrix form and vise versa.
-Understand and perform matrix operations.
-Finding the inverse of a square matrix.
-Finding the determinate of a square matrix.
- Use matrices and the determinate to solve real-world problems.

Critical Areas & Content

I) Multivariable Linear Systems
- Section 7-3

II) Matrices and Systems of Equations
- Section 7-4

III) Operations with Matrices
- Section 7-5

IV) The Inverse of a Square Matrix
- Section 7-6

V) The Determinant of a Square Matrix
- Section 7-7

VI) Applications of Matrices
- Section 7-8

Content Standards

N-VM 6
N-VM 7
N-VM 8
N-VM 9
N-VM 10
N-VM 11
N-VM 12