CRITICAL AREAS

Geometry

The fundamental purpose of the Geometry course is to formalize and extend students' geometric experiences from the middle grades. This course includes standards from the conceptual categories of Geometry and Statistics and Probability. Some standards are repeated in multiple higher mathematics courses; therefore instructional notes, which appear in brackets, indicate what is appropriate for study in this particular course.

In this Geometry course, students explore more complex geometric situations and deepen their explanations of geometric relationships, presenting and hearing formal mathematical arguments. Important differences exist between this course and the historical approach taken in geometry classes. For example, transformations are emphasized in this course.

For the Geometry course, instructional time should focus on six critical areas: (1) establish criteria for congruence of triangles based on rigid motions; (2) establish criteria for similarity of triangles based on dilations and proportional reasoning; (3) informally develop explanations of circumference, area, and volume formulas; (4) apply the Pythagorean Theorem to the coordinate plan; (5) prove basic geometric theorems; and (6) extend work with probability.

(1) Students have prior experience with drawing triangles based on given measurements and performing rigid motions including translations, reflections, and rotations. They have used these to develop notions about what it means for two objects to be congruent. In this course, students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They use triangle congruence as a familiar foundation for the development of formal proof. Students prove theorems—using a variety of formats including deductive and inductive reasoning and proof by contradiction—and solve problems about triangles, quadrilaterals, and other polygons. They apply reasoning to complete geometric constructions and explain why they work.

(2) Students apply their earlier experience with dilations and proportional reasoning to build a formal understanding of similarity. They identify criteria for similarity of triangles, use similarity to solve problems, and apply similarity in right triangles to understand right triangle trigonometry, with particular attention to special right triangles and the Pythagorean Theorem. Students derive the Laws of Sines and Cosines in order to find missing measures of general (not necessarily right) triangles, building on their work with quadratic equations done in Algebra I. They are able to distinguish whether three given measures (angles or sides) define 0, 1, 2, or infinitely many triangles.

(3) Students' experience with three-dimensional objects is extended to include informal explanations of circumference, area, and volume formulas. Additionally, students apply their knowledge of two-dimensional shapes to consider the shapes of cross-sections and the result of rotating a two-dimensional object about a line.

(4) Building on their work with the Pythagorean Theorem to find distances, students use the rectangular coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals, and slopes of parallel and perpendicular lines, which relates back to work done in the Algebra I course. Students continue their study of quadratics by connecting the geometric and algebraic definitions of the parabola.

(5) Students prove basic theorems about circles, with particular attention to perpendicularity and inscribed angles, in order to see symmetry in circles and as an application of triangle congruence criteria. They study relationships among segments on chords, secants, and tangents as an application of similarity. In the Cartesian coordinate system, students use the distance formula to write the equation of a circle when given the radius and the coordinates of its center. Given an equation of a circle, they draw the graph in the coordinate plane, and apply techniques for solving quadratic equations—which relates back to work done in the Algebra I course—to determine intersections between lines and circles or parabolas and between two circles.

(6) Building on probability concepts that began in the middle grades, students use the language of set theory to expand their ability to compute and interpret theoretical and experimental probabilities for compound events, attending to mutually exclusive events, independent events, and conditional probability. Students should make use of geometric probability models wherever possible. They use probability to make informed decisions.

The Standards for Mathematical Practice complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years.

Mathematical Practice	Explanation and Examples
	MPs aligned to EL/ELD and NGSS: MP 1, 3, 6
MP.1 Make sense of problems and persevere in solving them.	Geometry students learn that often patience is required to fully understand what a problem is asking. They discern between what information is useful, and what is not. Students construct accurate diagrams of geometry problems to help make sense of them. They organize their work so that others can follow their reasoning, e.g., in proofs.
MP.2 Reason Abstractly and quantitatively	Students understand that the coordinate plane can be used to represent geometric shapes and transformations and therefore connect their understanding of number and algebra to geometry.
MP.3 Construct viable arguments and critique the reasoning of others	Students construct proofs of geometric theorems. They write coherent logical arguments and understand that each step in a proof must follow from the last, justified with a previously accepted or proven result.
MP.4 Model with mathematics	Students apply their new mathematical understanding to real world problems. Students discover new mathematics through experimentation and solving contextual problems. They learn how transformational geometry and trigonometry can be used to model the physical world. Mathematical Practice Standard 4 holds a special place throughout the higher mathematics curriculum, as Modeling is considered its own conceptual category. Though the Modeling category has no specific standards listed within it, the idea of using mathematics to model the world pervades all higher mathematics courses and should hold a high place in instruction. Readers will see some standards marked with a star symbol (★) to indicate that they are *modeling standards*, that is, they present an opportunity for applications to real-world modeling situations more so than other standards.
MP.5 Use appropriate tools strategically	Students make use of visual tools for representing geometry, such as simple patty paper or transparencies, or dynamic geometry software.
MP.6 Attend to precision	Students develop and use precise definitions of geometric terms. They verify that a specific shape has certain properties justifying its categorization (e.g. a rhombus as opposed to a quadrilateral).
MP.7 Look for and make use of structure	Students construct triangles in quadrilaterals or other shapes and use congruence criteria of triangles to justify results about those shapes.
MP.8 Look for and express regularity in repeated reasoning	Students explore rotations, reflections and translations, noticing that certain attributes of different shapes remain the same (e.g. parallelism, congruency, orientation) and develop properties of transformations by generalizing these observations.

2013:

Geometry

Congruence

- Experiment with transformations in the plane.
- Understand congruence in terms of rigid motions.
- Prove geometric theorems.
- Make geometric constructions.

Similarity, Right Triangles, and Trigonometry

- Understand similarity in terms of similarity transformations.
- Prove theorems involving similarity.
- Define trigonometric ratios and solve problems involving right triangles.
- Apply trigonometry to general triangles.

Circles

- Understand and apply theorems about circles.
- Find arc lengths and area of sectors of circles.

Expressing Geometric Properties with Equations

- Translate between the geometric description and the equation for a conic section.
- Use coordinates to prove simple geometric theorems algebraically.

Geometric Measurement and Dimension

- Explain volume formulas and use them to solve problems.
- Visualize relationships between two-dimensional and three-dimensional objects

Modeling with Geometry

- Apply geometric concepts in modeling situations.

Statistics and Probability

Conditional Probability and the Rules of Probability

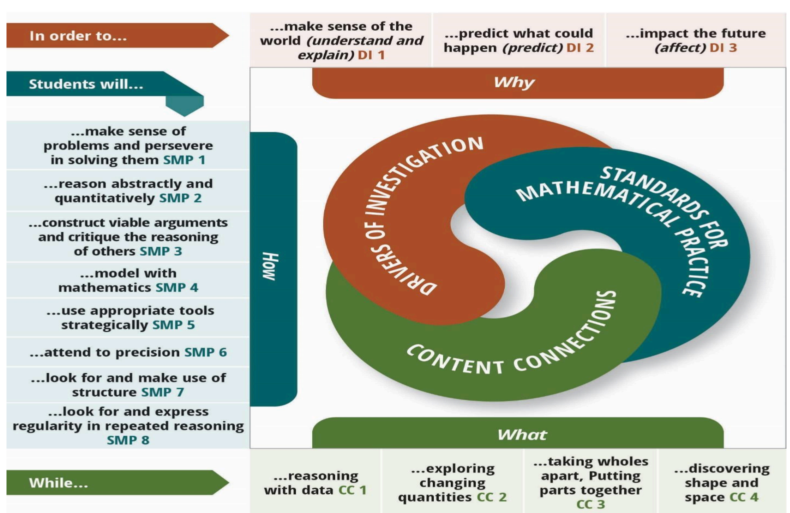
- Understand independence and conditional probability and use them to interpret data.
- Use the rules of probability to compute probabilities of compound event in a uniform probability model.

Using Probability to Make Decisions

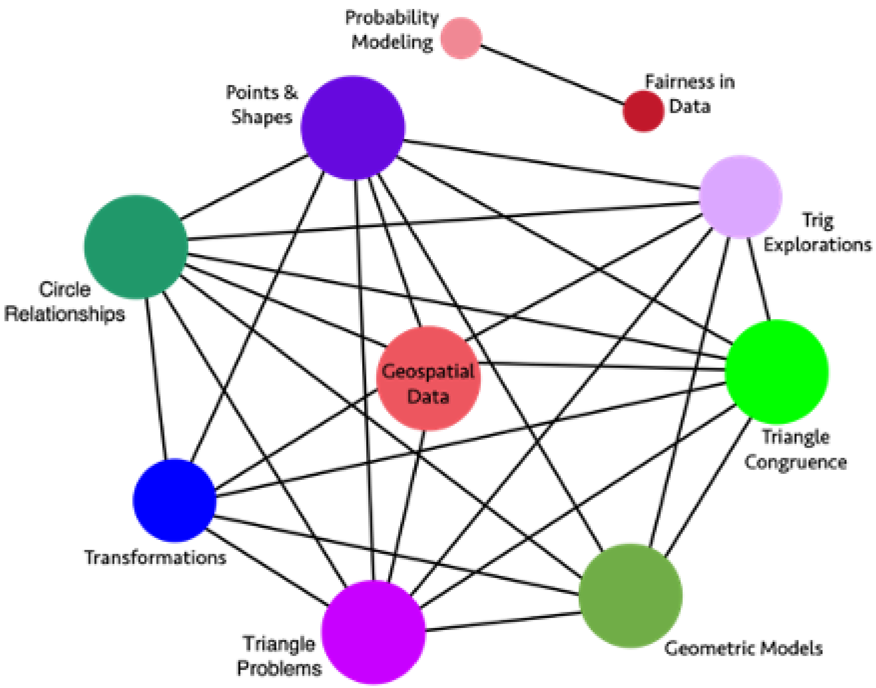
- Use probability to evaluate outcomes of decisions.

Math Core 8 Milestones as background content knowledge and expectations with which students enter Algebra I
Algebra Warm Ups for Geometry Teachers ~ Keeping it Fresh!

2023:



GEOMETRY BIG IDEAS from 2023 CA MATH FRAMEWORK chapter 8



The graphic illustrates the connections and relationships of some high school geometry mathematics concepts. Direct connections include the following:

- Probability Modeling directly connects to: Fairness in Data
- Fairness in Data directly connects to: Probability Modeling
- Trig Explorations directly connects to: Triangle Congruence, Geometric Models, Triangle Problems, Geospatial Data, Circle Relationships, Points & Shapes
- Triangle Congruence directly connects to: Geometric Models, Triangle Problems, Transformations, Geospatial Data, Circle Relationships, Points & Shapes, Trig Explorations
- Geometric Models directly connects to: Triangle Problems, Transformations, Circle Relationships, Points & Shapes, Trig Explorations, Triangle Congruence
- Triangle Problems directly connects to: Geometric Models, Triangle Congruence, Transformations, Geospatial Data, Circle Relationships, Points & Shapes, Trig Explorations
- Transformations directly connects to: Geometric Models, Triangle Problems, Triangle Congruence, Geospatial Data, Circle Relationships, Points & Shapes
- Circle Relationships directly connects to: Geometric Models, Triangle Problems, Transformations, Geospatial Data, Triangle Congruence, Points & Shapes, Trig Explorations

· Points & Shapes directly connects to: Geometric Models, Triangle Problems, Transformations, Geospatial Data, Circle Relationships, Triangle Congruence, Trig Explorations

· Geospatial Data: Triangle Problems, Transformations, Triangle Congruence, Circle Relationships, Points & Shapes, Trig Explorations

CONTENT CONNECTIONS ~ BIG IDEAS ~ CONTENT STANDARDS

CC1~reasoning with data   CC2~exploring changing quantities

CC3~taking wholes apart, putting parts together   CC4~discovering shape and space

Big Idea	Content Connection	Geometry Content Standards
Probability Modeling	Reasoning with Data	**S-CP.1, S-CP.2, S-CP.3, S-CP.4, S-CP.5, S-IC.1, S-IC.2, S-IC.3, S-MD.6, S-MD.7:** Explore and compare independent and conditional probabilities, interpreting the output in terms of the model. Construct and interpret two-way frequency tables of data as a sample space to determine if the events are independent and use the data to approximate conditional probabilities. Examples of topics include product and medical testing, and player statistics in sports.
Fairness in Data	Reasoning with Data	**S-MD.6, S-MD.7:** Determine fairness and make decisions based on evaluation of outcomes. Allow students to explore fairness by researching topics of interest, analyzing data from two-way tables. Provide opportunities for students to make meaningful inference, and communicate their findings to community or other stakeholders.
Geospatial Data	Reasoning with Data	**G-MG.1, G-MG.2, G-MG.3, F-LE.6, G-GPE.4, G-GPE.6, G-SRT.5, G-CO.1, G-CO.2, G-CO.12, G-C.2, G-C.5:** Explore geospatial data that represent either locations (e.g., maps) or objects (e.g., patterns of people's faces, road objects for driverless cars), and connect to geometric equations and properties of common shapes. Demonstrate how a computer can measure the distance between two points using geometry, and then account for constraints (e.g., distance and then roads for directions) and multiple points with triangulation. Model what shapes and geometric relationships are most appropriate for different situations.
Trig Explorations	Exploring Changing Quantities	**G-SRT.1, G-SRT.2, G-SRT.3, G-SRT.5, G-SRT.9, G-SRT.10, G-SRT.11, G-GPE.7. G-C.2, G-C.4:** Investigate properties of right triangle similarity and congruence and the relationships between sine, cosine, and tangent; explore the relationship between sine and cosine of complementary angles, and apply that knowledge to problem solving situations. Students recognize the role similarity plays in establishing trigonometric functions, and they use trigonometric functions to investigate situations. Using dynamic geometric software students investigate similarity and trigonometric identities to derive the Laws of Sines and Cosines and use the laws to solve problems.

Big Idea	Content Connection	Geometry Content Standards
Triangle Problems	Exploring Changing Quantities	**G-SRT.4, G-SRT.5, G-SRT.6, G-SRT.8, G-C.2, G-C.4, G-CO.12:** Understand and use congruence and similarity when solving problems involving triangles, including trigonometric ratios. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems using dynamic geometric software.
Points and Shapes	Exploring Changing Quantities	**G-GPE.1, G-GPE.2, G-GPE.4, G-GPE.5, G-GPE.6, G-GPE.7, G-CO.1, G-CO.12, G-C.2, G-C.4:** Solve problems involving geometric shapes in the coordinate plane using dynamic geometric software to apply the distance formula, Pythagorean Theorem, slope, and similarity rules in solving problems. • Investigate equations of circles and how coefficients in the equations correspond to the location and radius of the circles. Find areas and perimeters of triangles and rectangles in the coordinate plane.
Transformations	Taking Wholes Apart, Putting Parts Together and Discovering Shape and Space	**G-CO.1, G-CO.3, G-CO.4, G-CO.5, G-CO.12**: Understand rotations, reflections, and translations of regular polygons, quadrilaterals, angels, circles, and line segments. Identify transformations, through investigation, that move a figure back onto itself, using that process to prove congruence.
Triangle Congruence	Discovering Shape and Space and Exploring Changing Quantities and Taking Wholes Apart, Putting Parts Together	**G-CO.1, G-CO.2, G-CO.7, G-CO.8, G-CO.9, G-CO.10, G-CO.11, G-CO.12, G-CO.13, G-SRT.5:** Investigate triangles and their congruence over rigid transformations verifying findings using triangle congruence theorems (ASA, SSS, SAS, AAS, and HL) and other geometric properties, including vertical angles, angles created by transversals across parallel lines, and bisectors.
Circle Relationships	Exploring Changing Quantities and Discovering Shape and Space	**G-C.1, G-C.2, G-C.3, G-C.4, G-CO.1, G-CO.12, G-CO.13, G-GPE.1:** Investigate similarity in circles and relationships between angle measures and segments, including inscribed angles, radii, chords, central angles, inscribed angles, circumscribed angles, and tangent lines using dynamic geometric software.
Geometric Models	Discovering Shape and Space	**G-GMD.1, G-GMD.3, G-GMD.4, G-GMD.5, G-MG.1, G-MG.2, G-MG.3, G-SRT.5, G-CO.12, G-C.2, G-C.4:** Apply geometric concepts in modeling situations to solve design problems using dynamic geometric software. • Investigate 3-D shapes and their cross sections. • Use volume, area, circumference, and perimeter formulas. • Understand and apply Cavalieri's principle. • Investigate and apply scale factors for length, area, and volume.

Drivers of Investigation. Unifying reasons that both elicit curiosity and provide the motivation for deeply engaging with authentic mathematics.

D1~make sense of the world (understand and explain)

D2~predict what could happen (predict)

D3~impact the future (affect)