# **CRITICAL AREAS**







# Algebra II

Building on their work with linear, quadratic, and exponential functions, students extend their repertoire of functions to include logarithmic, polynomial, rational, and radical functions in the Algebra II course. This course includes standards from the conceptual categories of Number and Quantity, Algebra, Functions, Geometry, and Statistics and Probability. Some standards are repeated in multiple higher mathematics courses; therefore instructional notes, which appear in brackets, indicate what is appropriate for study in this particular course. Standards that were limited in Algebra I no longer have those restrictions in Algebra II. Students work closely with the expressions that define the functions, competently manipulate algebraic expressions, and continue to expand and hone their abilities to model situations and to solve equations, including solving quadratic equations over the set of complex numbers and solving exponential equations using the properties of logarithms.

For the Algebra II course, instructional time should focus on four critical areas: (1) relate arithmetic of rational expressions to arithmetic of rational numbers; (2) expand understandings of functions and graphing to include trigonometric functions; (3) synthesize and generalize functions and extend understanding of exponential functions to logarithmic functions; and (4) relate data display and summary statistics to probability and explore a variety of data collection methods.

- (1) A central theme of this Algebra II course is that the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers. Students explore the structural similarities between the system of polynomials and the system of integers. They draw on analogies between polynomial arithmetic and base-ten computation, focusing on properties of operations, particularly the distributive property. Connections are made between multiplication of polynomials with multiplication of multi-digit integers, and division of polynomials with long division of integers. Students identify zeros of polynomials, including complex zeros of quadratic polynomials, and make connections between zeros of polynomials and solutions of polynomial equations. The Fundamental Theorem of Algebra is examined.
- (2) Building on their previous work with functions and on their work with trigonometric ratios and circles in the Geometry course, students now use the coordinate plane to extend trigonometry to model periodic phenomena.
- (3) Students synthesize and generalize what they have learned about a variety of function families. They extend their work with exponential functions to include solving exponential equations with logarithms. They explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of the underlying function. They identify appropriate types of functions to model a situation, they adjust parameters to improve the model, and they compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit. The description of modeling as "the process of choosing and using mathematics and statistics to analyze empirical situations, to understand them better, and to make decisions" is at the heart of this Algebra II course. The narrative discussion and diagram of the modeling cycle should be considered when knowledge of functions, statistics, and geometry is applied in a modeling context.
- (4) Students see how the visual displays and summary statistics they learned in earlier grades relate to different types of data and to probability distributions. They identify different ways of collecting data—including sample surveys, experiments, and simulations—and consider how randomness and careful design affect the conclusions that can be drawn.

The Standards for Mathematical Practice complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years.

Mathematical Practice	Explanation and Examples
	MPs aligned to EL/ELD and NGSS: MP 1, 3, 6
MP.1 Make sense of problems and persevere in solving them.	Algebra II students learn that often patience is required to fully understand what a problem is asking. They discern between what information is useful, and what is not. They expand their repertoire of expressions and functions that can be used to solve problems. Students apply their understanding of various functions to real-world problems. They approach complex mathematics problems and break them down into smaller-sized chunks and synthesize the results when presenting solutions.
MP.2 Reason Abstractly and quantitatively	Students deepen their understanding of variable, for example, by understanding that changing the values of the parameters in the expression $Asin (Bx + C) + D$ has consequences for the graph of the function. They interpret these parameters in a real world context.
MP.3 Construct viable arguments and critique the reasoning of others	Students continue to reason through the solution of an equation and justify their reasoning to their peers. Students defend their choice of a function to model a real world situation.
MP.4 Model with mathematics	Students apply their new mathematical understanding to real world problems, making use of their expanding repertoire of functions in modeling. Students also discover mathematics through experimentation and examining patterns in data from real world contexts.
Bundle - Contraction	Mathematical Practice Standard 4 holds a special place throughout the higher mathematics curriculum, as Modeling is considered its own conceptual category. Though the Modeling category has no specific standards listed within it, the idea of using mathematics to model the world pervades all higher mathematics courses and should hold a high place in instruction. Readers will see some standards marked with a star symbol ( $\bigstar$ ) to indicate that they are <i>modeling standards</i> , that is, they present an opportunity for applications to real-world modeling situations more so than other standards.
MP.5 Use appropriate tools strategically	Students continue to use graphing technology to deepen their understanding of the behavior of polynomial, rational, square root, and trigonometric functions.
MP.6 Attend to precision	Students make note of the precise definition of <i>complex number</i> , understanding that real numbers are a subset of the complex numbers. They pay attention to units in real world problems and use unit analysis as a method for verifying their answers.
MP.7 Look for and make use of structure	Students see the operations of the complex numbers as extensions of the operations for real numbers. They understand the periodicity of sine and cosine and use these functions to model periodic phenomena.
MP.8 Look for and express regularity in repeated reasoning	Students observe patterns in geometric sums, e.g. that the first several sums of the form $\sum_{k=0}^{n} 2^{k}$ can be written: $1 = 2^{1} - 1$ ; $1 + 2 = 2^{2} - 1$ ; $1 + 2 + 4 = 2^{3} - 1$ ; $1 + 2 + 4 + 8 = 2^{4} - 1$ , and use this observation to make a conjecture about any such sum.

# 2013:

#### Number and Quantity

#### The Complex Number System

- Perform arithmetic operations with complex numbers
- Use complex numbers in polynomial identities and equations.

#### Algebra

#### Seeing Structure in Expressions

- Interpret the structure of expressions.
- Write expressions in equivalent forms to solve problems.

#### Arithmetic with Polynomials and Rational Expressions

- Perform arithmetic operations on polynomials.
- Understand the relationship between zeros and factors of polynomials.
- Use polynomial identities to solve problems.
- Rewrite rational expressions

# **Creating Equations**

• Create equations that describe numbers or relationships.

#### Reasoning with Equations and Inequalities

- Understand solving equations as a process of reasoning and explain the reasoning.
- Solve equations and inequalities in one variable.
- Represent and solve equations and inequalities graphically.

# Functions

# Interpreting Functions

- Interpret functions that arise in applications in terms of the context.
- Analyze functions using different representations.

# **Building Functions**

- Build a function that models a relationship between two quantities.
- Build new functions from existing functions.

# Linear, Quadratic, and Exponential Models

• Construct and compare linear, quadratic, and exponential models and solve problems.

# **Trigonometric Functions**

- Extend the domain of trigonometric functions using the unit circle.
- Model periodic phenomena with trigonometric functions.
- Prove and apply trigonometric identities.

# Geometry

# **Expressing Geometric Properties with Equations**

• Translate between the geometric description and the equation for a conic section.

# Statistics and Probability

# Interpreting Categorical and Quantitative Data

• Summarize, represent, and interpret data on a single count or measurement variable.

# Making Inferences and Justifying Conclusions

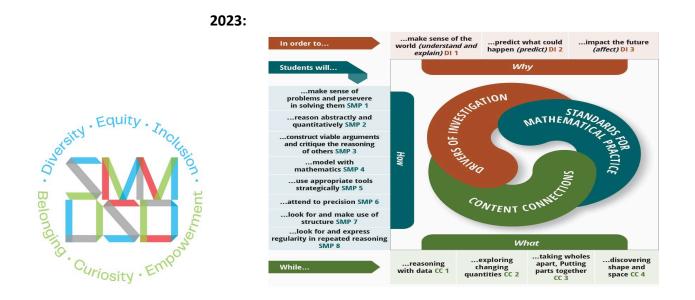
- Understand and evaluate random processes underlying statistical experiments.
- Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

# Using Probability to Make Decisions

• Use probability to evaluate outcomes of decisions.

#### Math Core 8 Milestones as background content knowledge and expectations with which students enter Algebra I Algebra Warm Ups for Geometry Teachers, and others... ~ Keeping it Fresh!

Conserve Envoyed



# ALGEBRA II BIG IDEAS from 2023 CA MATH FRAMEWORK chapter 8

The standards in the Algebra II course come from the conceptual categories of Modeling, Functions, Number and Quantity, Algebra, and Statistics and Probability. Building on their work with linear, quadratic, and exponential functions, students in Algebra II extend their repertoire of functions to include polynomial, rational, and radical functions.

Students work closely with the expressions that define the functions and continue to expand and hone their abilities to model situations and to solve equations, including solving quadratic equations over the set of complex numbers and solving exponential equations using the properties of logarithms. Based on their previous work with functions, and on their work with trigonometric ratios and circles in geometry, students now use the coordinate plane to extend trigonometry to model periodic phenomena. They explore the effects of transformations on graphs of diverse functions, including functions arising in applications, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of underlying function. They identify appropriate types of functions to model a situation, adjust parameters to improve the model, and compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit.

# **CONTENT CONNECTIONS ~ BIG IDEAS ~ CONTENT STANDARDS**

CC1<sup>~</sup>reasoning with data CC2<sup>~</sup>exploring changing quantities CC3<sup>~</sup>taking wholes apart, putting parts together CC4<sup>~</sup>discovering shape and space

Drivers of Investigation. Unifying reasons that both elicit curiosity and provide the motivation for deeply engaging with authentic mathematics.

D1~make sense of the world (understand and explain)

D2~predict what could happen (predict)

D3~impact the future (affect)

