

CRITICAL AREAS



Grade 6



In grade 6, instructional time should focus on four critical areas: (1) connecting ratio and rate to whole number multiplication and division, and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting, and using expressions and equations; and (4) developing understanding of statistical thinking.

- (1) Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students solve a wide variety of problems involving ratios and rates.
- (2) Students use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems. Students extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.
- (3) Students understand the use of variables in mathematical expressions. They write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems. Students understand that expressions in different forms can be equivalent, and they use the properties of operations to rewrite expressions in equivalent forms. Students know that the solutions of an equation are the values of the variables that make the equation true. Students use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. Students construct and analyze tables, such as tables of quantities that are in equivalent ratios, and they use equations (such as $3x = y$) to describe relationships between quantities.
- (4) Building on and reinforcing their understanding of number, students begin to develop their ability to think statistically. Students recognize that a data distribution may not have a definite center and that different ways to measure center yield different values. The median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students recognize that a measure of variability (interquartile range or mean absolute deviation) can also be useful for summarizing data because two very different sets of data can have the same mean and median yet be distinguished by their variability. Students learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps, and symmetry, considering the context in which the data were collected.

Students in grade 6 also build on their work with area in elementary school by reasoning about relationships among shapes to determine area, surface area, and volume. They find areas of right triangles, other triangles, and special quadrilaterals by decomposing these shapes, rearranging or removing pieces, and relating the shapes to rectangles. Using these methods, students discuss, develop, and justify formulas for areas of triangles and parallelograms. Students find areas of polygons and surface areas of prisms and pyramids by decomposing them into pieces whose area they can determine. They reason about right rectangular prisms with fractional side lengths to extend formulas for the volume of a right rectangular prism to fractional side lengths. They prepare for work on scale drawings and constructions in grade 7 by drawing polygons in the coordinate plane.

<u>Mathematical Practice</u>	Explanation and Examples
	MPs aligned to EL/ELD and NGSS: MP 1, 3, 6
MP.1 Make sense of problems and persevere in solving them.	In grade 6, students solve real world problems through the application of algebraic and geometric concepts. These problems involve ratio, rate, area, and statistics. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, “What is the most efficient way to solve the problem?”, “Does this make sense?”, and “Can I solve the problem in a different way?” Students can explain the relationships between equations, verbal descriptions, tables and graphs. Mathematically proficient students check answers to problems using a different method.
MP.2 Reason Abstractly and quantitatively	In grade 6, students represent a wide variety of real world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.
MP.3 Construct viable arguments and critique the reasoning of others	In grade 6, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (e.g., box plots, dot plots, histograms). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like “How did you get that?”, “Why is that true?” and “Does that always work?” They explain their thinking to others and respond to others’ thinking.
MP.4 Model with mathematics	In grade 6, students model problem situations symbolically, graphically, tabularly, and contextually. Students form expressions, equations, or inequalities from real world contexts and connect symbolic and graphical representations. Students begin to explore covariance and represent two quantities simultaneously. Students use number lines to compare numbers and represent inequalities. They use measures of center and variability and data displays (i.e. box plots and histograms) to draw inferences about and make comparisons between data sets. Students need many opportunities to connect and explain the connections between the different representations. They should be able to use any of these representations as appropriate to a problem context.
MP.5 Use appropriate tools strategically	Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in grade 6 may decide to represent figures on the coordinate plane to calculate area. Number lines are used to understand division and to create dot plots, histograms, and box plots to visually compare the center and variability of the data. Additionally, students might use physical objects or applets to construct nets and calculate the surface area of three-dimensional figures.
MP.6 Attend to precision	In grade 6, students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to rates, ratios, geometric figures, data displays, and components of expressions, equations, or inequalities. When using ratio reasoning in solving problems, students are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. Students also learn to determine an appropriate degree of precision when working with rational numbers in a situational problem.
MP.7 Look for and make use of structure	Students routinely seek patterns or structures to model and solve problems. For instance, students recognize patterns that exist in ratio tables recognizing both the additive and multiplicative properties. Students apply properties to generate equivalent expressions (e.g., $6 + 2x = 3(2 + x)$ by distributive property) and solve equations (e.g., $2c + 3 = 15$, $2c = 12$ by subtraction property of equality, $c=6$ by division property of equality). Students compose and

	decompose two- and three-dimensional figures to solve real world problems involving area and volume.
MP.8 Look for and express regularity in repeated reasoning	In grade 6, students use repeated reasoning to understand algorithms and make generalizations about patterns. During multiple opportunities to solve and model problems, they may notice that $a/b \div c/d = ad/bc$ and construct other examples and models that confirm their generalization. Students connect place value and their prior work with operations to understand algorithms to fluently divide multi-digit numbers and perform all operations with multi-digit decimals. Students informally begin to make connections between covariance, rates, and representations showing the relationships between quantities.

2013:

The Number Sense

- Apply and extend previous understandings of multiplication and division to divide fractions by fractions.
- Compute fluently with multi-digit numbers and find common factors and multiples.
- Apply and extend previous understandings of numbers to the system of rational numbers.

Ratios and Proportional Relationships

- Understand ratio concepts and use ratio reasoning to solve problems.

Geometry

- Solve real-world and mathematical problems involving area, surface area, and volume.

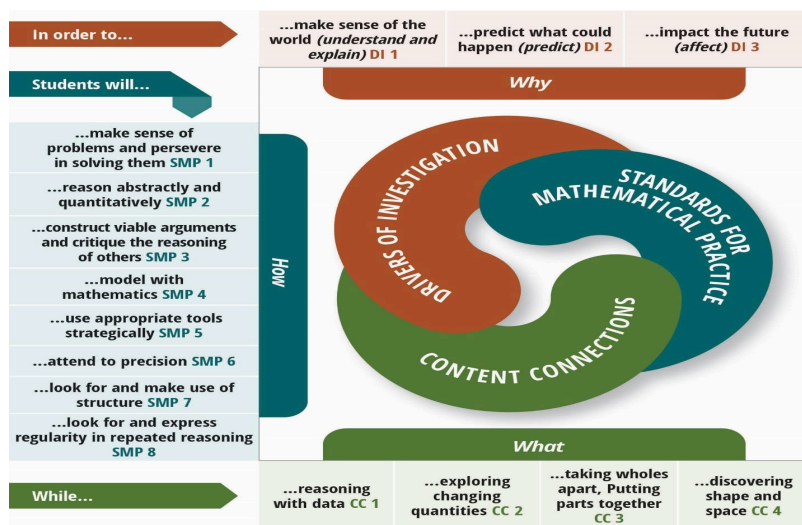
Expressions and Equations

- Apply and extend previous understandings of arithmetic to algebraic expressions
- Reason about and solve one-variable equations and inequalities.
- Represent and analyze quantitative relationships between dependent and independent variables.

Statistics and Probability

- Develop understanding of statistical variability.
- Summarize and describe distributions.

2023:



MATH CORE SIX **BIG IDEAS** from 2023 CA MATH FRAMEWORK chapter 7



- Variability in Data directly connects to: The Shape of Distributions, Relationships Between Variables
- The Shape of Distributions directly connects to: Relationships Between Variables, Variability in Data
- Fraction Relationships directly connects to: Patterns Inside Numbers, Generalizing with Multiple Representations, Model the World, Relationships Between Variables
- Patterns Inside Numbers directly connects to: Fraction Relationships, Generalizing with Multiple Representations, Model the World, Relationships Between Variables
- Generalizing with Multiple Representations directly connects to: Patterns Inside Numbers, Fraction Relationships, Model the World, Relationships Between Variables, Nets & Surface Area, Graphing Shapes
- Model the World directly connects to: Fraction Relationships, Relationships Between Variables, Patterns Inside Numbers, Generalizing with Multiple Representations, Graphing Shapes
- Graphing Shapes directly connects to: Model the World, Generalizing with Multiple Representations, Relationships Between Variables, Distance & Direction, Nets & Surface Area
- Nets & Surface Area directly connects to: Graphing Shapes, Generalizing with Multiple Representations, Distance & Direction
- Distance & Direction directly connects to: Graphing Shapes, Nets & Surface Area

Relationships Between Variables directly connects to: Variability in Data, The Shape of Distributions, Fraction Relationships, Patterns Inside Numbers, Generalizing with Multiple Representations, Model the World, Graphing Shapes



CONTENT CONNECTIONS ~ BIG IDEAS ~ CONTENT STANDARDS

CC1~reasoning with data CC2~exploring changing quantities

CC3~taking wholes apart, putting parts together CC4~discovering shape and space

Content Connection	Big Idea	Grade Six Content Standards
Reasoning with Data	Variability in Data	SP.1, SP.5, SP.4: Investigate real world data sources, ask questions of data, start to understand variability - within data sets and across different forms of data, consider different types of data, and represent data with different representations.
Reasoning with Data	The Shape of Distributions	SP.2, SP.3, SP.5: Consider the distribution of data sets - look at their shape and consider measures of center and variability to describe the data and the situation which is being investigated.
Exploring Changing Quantities	Fraction Relationships	NS.1, RP.1, RP.3: Understand fractions divided by fractions, thinking about them in different ways (e.g., how many $\frac{1}{3}$ are inside $\frac{2}{3}$?), considering the relationship between the numerator and denominator, using different strategies and visuals. Relate fractions to ratios and percentages.
Exploring Changing Quantities	Patterns inside Numbers	NS.4, RP.3: Consider how numbers are made up, exploring factors and multiples, visually and numerically.
Exploring Changing Quantities	Generalizing with Multiple Representations	EE.6, EE.2, EE.7, EE.3, EE.4, RP.1, RP.2, RP.3: Generalize from growth or decay patterns, leading to an understanding of variables. Understand that a variable can represent a changing quantity or an unknown number. Analyze a mathematical situation that can be seen and solved in different ways and that leads to multiple representations and equivalent expressions. Where appropriate in solving problems, use unit rates.
Exploring Changing Quantities	Relationships Between Variables	EE.9, EE.5, RP.1, RP.2, RP.3, NS.8, SP.1, SP.2: Use independent and dependent variables to represent how a situation changes over time, recognizing unit rates when it is a linear relationship. Illustrate the relationship using tables, 4 quadrant graphs and equations, and understand the relationships between the different representations and what each one communicates.

Content Connection	Big Idea	Grade Six Content Standards
Taking Wholes Apart, Putting Parts Together	Model the World	NS.3, NS.2, NS.8, RP.1, RP.2, RP.3: Solve and model real world problems. Add, subtract, multiply, and divide multi-digit numbers and decimals, in real-world and mathematical problems - with sense making and understanding, using visual models and algorithms.
Taking Wholes Apart, Putting Parts Together and Discovering Shape and Space	Nets and Surface Area	EE.1, EE.2, G.4, G.1, G.2, G.3: Build and decompose 3-D figures using nets to find surface area. Represent volume and area as expressions involving whole number exponents.
Discovering Shape and Space	Distance and Direction	NS.5, NS.6, NS.7, G.1, G.2, G.3, G.4: Students experience absolute value on numbers lines and relate it to distance, describing relationships, such as order between numbers using inequality statements.
Discovering Shape and Space	Graphing Shapes	G.3, G.1, G.4, NS.8, EE.2: Use coordinates to represent the vertices of polygons, graph the shapes on the coordinate plane, and determine side lengths, perimeter, and area.

Drivers of Investigation. Unifying reasons that both elicit curiosity and provide the motivation for deeply engaging with authentic mathematics.

D1~make sense of the world (understand and explain)

D2~predict what could happen (predict)

D3~impact the future (affect)

